

Alpha Channeling in a Rotating Plasma

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The wave-particle α -channeling effect is generalized to include rotating plasma. Specifically, radio frequency waves can resonate with α particles in a mirror machine with $\mathbf{E} \times \mathbf{B}$ rotation to diffuse the α particles along constrained paths in phase space. Of major interest is that the α -particle energy, in addition to amplifying the RF waves, can directly enhance the rotation energy which in turn provides additional plasma confinement in centrifugal fusion reactors. An ancillary benefit is the rapid removal of alpha particles, which increases the fusion reactivity.

In magnetic mirror fusion devices, centrifugal forces can significantly enhance the magnetic confinement [1, 2, 3]. A radial electric field induces rapid $\mathbf{E} \times \mathbf{B}$ plasma rotation, leading to the centrifugal force that directly confines ions axially. Electrons are then confined axially through the ambipolar potential. The radial field thus not only enhances the plasma confinement, but also produces the necessary heating for the plasma, as injected cold neutral fuel atoms are seen as moving at the rotation velocity in the rotating frame. Lately there has been a renewed interest in this effect [4, 5, 6, 7], strengthened by recent findings of reduced turbulence due to sheared rotation [8, 9].

What we show here is that in a DT (deuterium-tritium) centrifugal fusion reactor, the energy of α -particles, the byproducts of the fusion reaction, might be advantageously induced to directly produce this rotation. The predicted effect relies on exploiting the population inversion of the birth distribution of α particles. This is a generalization of the *alpha channeling effect*, where injected wave energy can be amplified at the expense of the α -particle energy, with the alpha particles concomitantly removed as cold particles [10]. In tokamaks, if the wave energy is damped on ions, the fusion reactivity might be doubled [11]. Similar advantageous uses of α -channeling can be expected in mirror machines [12]. With several waves, a significant amount of the α -particle energy can be advantageously channeled in both tokamaks [13] and mirrors [14]. However, in previous considerations of α -channeling, the plasma was not rotating strongly.

In strongly rotating plasma, significant new effects can occur because there are two further reservoirs of particle energy, namely rotational and potential energy. For example, through a suitable choice of wave parameters, particles can now absorb wave energy yet cool in kinetic energy, with the excess energy being stored in potential energy. Alternatively, particle potential energy might be lost to wave energy with kinetic energy constant. These possibilities could not be achieved through particle manipulation in stationary systems, where the only coupling is between the kinetic energy with the wave energy. What is important for centrifugal mirror fusion is that radiofrequency waves can drive a radial α -particle current, with the dissipated power extracted from the α -particle birth

energy, thereby maintaining the radial potential which produces the necessary plasma rotation.

To derive the new effects, define the angular rotation frequency $\Omega_{\mathbf{E}} = \Omega_E \hat{z}$, so that the $\mathbf{E} \times \mathbf{B}$ drift velocity can be written as $\Omega_E \times \mathbf{r} = \mathbf{E} \times \mathbf{B}/B^2$. For simplicity, consider constant Ω_E (solid-body rotation). Although some aspects may vary with the rotation profile, the concept should be applicable to arbitrary profiles $\Omega(r)$. The electric and magnetic field in the rotating frame are [15],

$$\tilde{\mathbf{E}} = \mathbf{E} + \frac{m}{q} \Omega^2 \mathbf{r} + (\Omega \times \mathbf{r}) \times \mathbf{B}, \quad (1)$$

$$\tilde{\mathbf{B}} = \mathbf{B} + 2 \frac{m}{q} \Omega. \quad (2)$$

The second term in Eq. (1) produces the centrifugal force, and the second term in Eq. (2) is due to the Coriolis effect. For $\Omega = \Omega_E$, the first and third terms in Eq. (1) will cancel. However, there will still be drifts due to the centrifugal force. We define as Ω_E^* the unique frame of reference in which $\hat{\theta} \cdot \tilde{\mathbf{E}} \times \tilde{\mathbf{B}} = 0$ for the species of interest. Note that the magnetic moment is seen as invariant only in the frame rotating with frequency Ω_E^* . Our notational convention is to denote terms in this frame with a tilde.

Note that by flux conservation, $r^2/r_0^2 \propto \tilde{B}/\tilde{B}_0$. Thus for magnetic mirror ratio $R_m = \tilde{B}_m/\tilde{B}_0$, there is an effective confinement potential $\Phi_c = \frac{1}{2} m \Omega_E^{*2} r_0^2 (1 - R_m^{-1})$, which varies with the midplane particle radius r_0 . The loss-cone diagram is depicted in Fig. 1. The maximum confinement potential is $W_{E0w} (1 - R_m^{-1})$, where $W_{E0w} = \frac{1}{2} m \Omega_E^{*2} r_w^2$, and r_w is the midplane radius of the last field line not intersecting a wall.

Now consider a wave with frequency ω , parallel wave number k_{\parallel} and azimuthal mode number $n_{\theta} = k_{\theta} r$. Due to the rotation, the wave frequency in the rotating frame

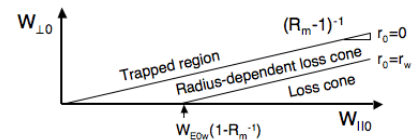


FIG. 1: The loss cone in (rotating) midplane energy coordinates for a rotating plasma, including centrifugal confinement.

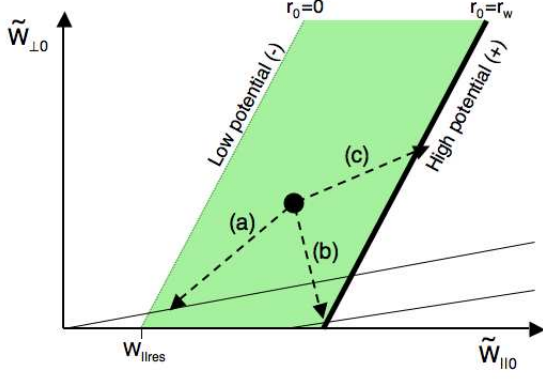


FIG. 2: The shaded region is the region of wave resonance, dependent on radius, for interaction with an RF wave at mirror ratio R_{rf} . The hatched lines depict three diffusion paths that would eject particles, using only perpendicular diffusion. Path (a) reduces both the kinetic and potential energy of the particle, path (b) increases potential energy but decreases kinetic energy, and path (c) increases both kinetic and potential energy. The energy balance is assumed by the wave.

will be $\tilde{\omega} = \omega - n_{\theta}\Omega_E^*$. The wave-particle resonance condition is then $\tilde{\omega} - k_{\parallel}v_{\parallel} = n\tilde{\Omega}_c$, where the resonance is at the n^{th} harmonic of the rotating-frame cyclotron frequency $\tilde{\Omega}_c = q\tilde{B}/m$ (q is the ion charge and m is its mass). The parallel velocity v_{\parallel} is independent of the rotating frame, and corresponds to energy $W_{\parallel res} = mv_{\parallel}^2/2$. Unlike in the stationary case, the related midplane parallel energy, $W_{\parallel 0 res}$, will not be constant across the radius of the device. For an RF region at mirror ratio $R_{rf} = \tilde{B}_{rf}/\tilde{B}_0$, the resonant parallel energy in rotating midplane coordinates is

$$W_{\parallel 0 res} = W_{\parallel res} + W_{E0} \left(1 - R_{rf}^{-1}\right), \quad (3)$$

where $W_{E0} = m\Omega_E^* r_0^2/2$. These resonant regions appear as the shaded region in Fig. 2.

Since the mirror system is axisymmetric, the diffusion paths will be the same as those for tokamaks [10],

$$dP_{\theta}/d\tilde{W} = n_{\theta}/\tilde{\omega}, \quad (4)$$

$$d\tilde{\mu}/d\tilde{W} = qn/m\tilde{\omega}, \quad (5)$$

where $\tilde{\mu} = m\tilde{v}_{\perp}^2/2\tilde{B}$ is the ion magnetic moment in the rotating frame; $\tilde{W} = \tilde{\mu}\tilde{B} + mv_{\parallel}^2/2$ is the kinetic energy in the rotating frame; and P_{θ} is the azimuthal canonical angular momentum (which is frame-independent).

The significant difference in the rotating frame is that the interaction of the particle with a wave at axial position z_{rf} changes the particle's perpendicular, parallel, and rotational kinetic energy, as well as its potential energy. The change in perpendicular energy may be written $\tilde{W}_{\perp}(z_{rf}) \rightarrow \tilde{W}_{\perp}(z_{rf}) + \Delta\tilde{W}_{\perp}$; the change in parallel energy, $\tilde{W}_{\parallel}(z_{rf}) \rightarrow \tilde{W}_{\parallel}(z_{rf}) + \Delta\tilde{W}_{\parallel}$; and the change in

rotational energy, $W_E(z_{rf}) \rightarrow W_E(z_{rf}) + \Delta W_E$. Thus the wave interaction, breaking the adiabatic invariance of $\tilde{\mu}$, gives stochastic kicks in $\Delta\tilde{W}_{\perp}$, $\Delta\tilde{W}_{\parallel}$, and ΔW_E .

The energy kicks are correlated through the properties of the wave. The relation between $\Delta\tilde{W}_{\perp}$ and $\Delta\tilde{W}_{\parallel}$ is found, by Eq. (5), to be $\Delta\tilde{W}_{\parallel} = \Delta\tilde{W}_{\perp}k_{\parallel}v_{\parallel}/(n\tilde{\Omega}_c)$. The radial excursion is determined in terms of the perpendicular energy change by Eq. (4), yielding $r\Delta r = \Delta\tilde{W}_{\perp}n_{\theta}/(m\tilde{\omega}\tilde{\Omega}_c)$. This then gives the rotational energy change, $\Delta W_E = m\Omega_E^* r\Delta r = \Delta\tilde{W}_{\perp}n_{\theta}\Omega_E^*/(\tilde{\omega}\tilde{\Omega}_c)$, and the potential energy change, $q\Delta\Phi = -qE\Delta r = n_{\theta}\Omega_E\Omega_c/(\tilde{\omega}\tilde{\Omega}_c)\Delta\tilde{W}_{\perp}$.

Using the adiabatic invariance of $\tilde{\mu}$, flux conservation ($r^2/r_0^2 \propto \tilde{B}/\tilde{B}_0$), and conservation of energy, we require

$$\Delta\tilde{W}_{\perp} + \Delta\tilde{W}_{\parallel} - \Delta W_E = R_{rf}^{-1}\Delta\tilde{W}_{\perp} + \Delta\tilde{W}_{\parallel 0} - R_{rf}\Delta W_E, \quad (6)$$

so that the changes in rotating midplane coordinates can be written as,

$$\Delta\tilde{W}_{\perp 0} = \Delta\tilde{W}_{\perp}/R_{rf}, \quad (7)$$

$$\Delta\tilde{W}_{\parallel 0} = \left[\frac{k_{\parallel}v_{\parallel}}{n\tilde{\Omega}_c} + (R_{rf} - 1) \frac{n_{\theta}\Omega_E^*}{\tilde{\omega}\tilde{\Omega}_c} + \left(1 - R_{rf}^{-1}\right) \right] \Delta\tilde{W}_{\perp}, \quad (8)$$

$$\Delta r_0 = \frac{R_{rf}n_{\theta}}{mr_0\tilde{\omega}\tilde{\Omega}_c} \Delta\tilde{W}_{\perp}. \quad (9)$$

As the particle diffuses in radius it also changes its rotation energy W_E . This will lead to a change in midplane parallel energy for $R_{rf} > 1$, as can be seen in Eq. (3). This is the source of the second term in brackets in Eq. (8). Note that the particle remains in resonance with the wave on its entire path in the limit $n_{\parallel} \rightarrow 0$.

With reference now to Fig. 2, note three ways particles might be extracted from a rotating mirror, with perpendicular diffusion only ($\Delta\tilde{W}_{\parallel} = 0$). The particle begins midway between the axis and wall of the device. Suppose the particle may be removed through the loss cone by path (a) at a low potential energy and a low kinetic energy. This requires the wave phase velocity in the rotating frame to be positive ($\tilde{v}_p = k_{\theta}/\tilde{\omega} > 0$). The same wave may be used to remove particles through the last flux surface at high kinetic and potential energy (shown by path (c)). The energy balance in each case is carried by the interacting wave. Path (b) describes a diffusion path where the particle is removed with less kinetic energy than at its birth but at a higher potential energy. The particle may be removed either through the loss cone or the last flux surface. This is the useful case for maintaining the radial electric field.

To calculate the *branching ratio* f_E , the ratio of energy going into the radial potential to the total energy change, consider that the change in rest-frame kinetic energy is

$$\Delta W = \left(\frac{\omega}{\tilde{\omega}} + \frac{k_{\parallel}v_{\parallel}}{n\tilde{\Omega}_c} \right) \Delta\tilde{W}_{\perp}, \quad (10)$$

giving the branching ratio

$$f_E = \frac{-n_\theta \Omega_E \Omega_c}{\omega \tilde{\Omega}_c + \tilde{\omega} k_\parallel v_\parallel / n - n_\theta \Omega_E \Omega_c}, \quad (11)$$

$$\approx \frac{-n_\theta \Omega_E}{\Omega_c + 2k_\parallel v_\parallel + 4\Omega_E}, \quad (12)$$

where the approximation in Eq. (12) uses the resonance condition $\tilde{\omega} = \tilde{\Omega}_c + k_\parallel v_\parallel$, with $\Omega_E, k_\parallel v_\parallel \ll \Omega_c$. If these conditions are sufficiently strong, the fraction of the total energy change provided to the radial electric field is $f_E \approx -n_\theta \Omega_E / \Omega_c$. In the case $f_E > 1$, the particle reduces its kinetic energy and simultaneously absorbs wave energy, which can be expected because the direction of the RF wave phase velocity in the rotating frame, $\tilde{v}_p = \tilde{\omega} / k_\theta$ is opposite that in the laboratory frame, $v_p = \omega / k_\theta$. Path (b) in Fig. 2 describes a diffusion path in which the wave will be amplified if $f_E < 1$ (not all kinetic energy is converted to potential), or damped if $f_E > 1$ (wave energy transferred to potential energy).

The waves necessary for a channeling effect have been calculated in the static mirror case [12, 14]. Two conditions were considered to be important. In order for the diffusion path to be favorable, it must connect a dense area of phase-space near the birth population to a less dense area of phase space near the loss boundary. In addition, it is advantageous that the α particle heating be limited above the birth energy. It was shown that waves with purely perpendicular diffusion ($\Delta W_\parallel = 0$, $n = 1$, $k_\parallel v_\parallel \ll \tilde{\Omega}_c$) and $T_i \ll W_{\parallel res} \ll W_{\alpha 0}$ satisfy the first requirement. In the rotating system, the results are the same: perpendicular diffusion with $W_{E0} < W_{\parallel res} \ll W_{\alpha 0}$ will provide connection to the velocity-space loss cone, and α particles will leave at low energy.

The limitation of the α particle heating along the diffusion path may be accomplished in two ways [14]. The first way is by diffusing energy-gaining particles inward, since the particle is restricted to $r \geq 0$. This is a strong limit to the energy gain, but requires large k_θ to limit the energy gain to a few MeV for large devices (see Eq. (9)). The second way to limit α -particle heating is by noting that the diffusion coefficient of a particle in a wave near harmonic n of the cyclotron frequency is proportional to $J_n^2(k_\theta \rho)$, where J_n is the Bessel function of the first kind and ρ is the gyroradius [16]. The diffusion coefficient will be nearly zero if $k_\theta \rho$ is equal to a zero of the Bessel function. For the first Bessel zero to be above the α -particle birth energy, $k_\theta < 3.8317 / \rho_{\alpha 0}$ is required.

In any case, it will improve efficiency to have particles gaining energy move inward, where they are less likely to be lost at high energy. Then by Eq. (9), n_θ must be negative. For an outward current to maintain the potential, Ω_E must be positive. This is also the preferred polarity for rotating mirror systems [1].

Efficient conversion of alpha birth energy to potential energy can be achieved by selecting n_θ such that $f_E \sim 1$.

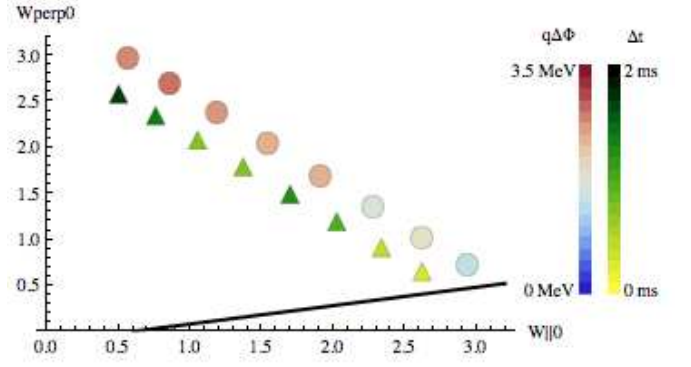


FIG. 3: Numerical simulation of alpha channeling. The circles indicate the amount of energy channeled into the radial potential. The triangles indicate the loss time. Both circles and triangles refer to the same set of alpha particles born at 3.5 MeV; their position indicates the ratio $W_{\perp 0} / W_{\parallel 0}$ at birth. Each point is the average of 20 single-particle simulations.

For $B_0 = 1$ T, this means that $-n_\theta \Omega_E \sim 50 \times 10^6 \text{ s}^{-1}$. An electric field of 50 kV/cm at $r = 1$ m produces $\Omega_E = 5 \times 10^6 / \text{s}$. To convert all of the α particle kinetic energy to electric potential energy, choose $n_\theta = 10$, or at $r = 1$ m, $k_\theta \approx 0.1 / \text{cm}$. This is also below the first Bessel function zero, which for 3.5 MeV α particles is near $k_\theta = 0.14 / \text{cm}$.

We simulated this alpha channeling effect in a rotating mirror plasma, by following the full equations of motion for 160 particles interacting with ten RF regions. The wave parameters $W_{\parallel res} = 750 \text{ keV}$, $k_\parallel = -0.03 / \text{cm}$, $E_{rf} = 30 \text{ kV/cm}$ were constant across each wave, but $n_\theta = 12 - 20$ and $R_{rf} = 1.2 - 4.5$ varied. The mirror ratio for the simulation was $R_m = 6$, and the device length was 20 m. The statistics were sufficient to estimate that 65% of the energy was channeled into the radial potential. Fig. 3 depicts, for these parameters, the relative effectiveness of energy extraction from alpha particles of different birth energies, as well as the relative time for extraction. The energy extraction needs to be completed before the collisional slowing down of the alpha particle.

The primary application of the α channeling effect proposed here is to use the energy of α particles to support the rotation of the plasma, which is the main power requirement in centrifugal fusion reactors; the heating of the fuel is then automatic since through ionization new fuel particles are born with a high kinetic energy in the rotating frame. Define the fusion energy gain, $Q \equiv P_f / P_{\text{circ}}$, as the ratio of fusion power to circulating power to maintain the rotation. Let η be the fraction of alpha particle power $P_f / 5$ that supports the rotation, then the fusion energy gain in the presence of the α channeling effect can be written as $Q_{AC} = Q / (1 - \eta Q / 5)$. Thus, if all of the α particle energy could be converted into rotation energy ($\eta = 1$), a reactor formerly operating at $Q = 5$ would become self-sustaining, requiring no external heating or energy input.

There are several considerations to address in choosing the branching ratio f_E in each wave region. In order to extract efficiently the energy of the alpha particles, they must be removed by diffusion (having rate proportional to the standing wave energy) before they are lost collisionally. Thus, the branching ratio should be set first to assure that the waves reach sufficient amplitude for collisionless diffusion. On the other hand, it is not beneficial to amplify the wave beyond what is needed to satisfy this criterion. The power going into the waves in other fusion devices is generally thought to be most effectively channeled into fuel ion heating [10, 12]; in contrast, the further heating of ions is not necessary in rotating mirrors, since they are born at high energy. To the extent that extra power is used to support the rotation, beyond the necessary rotation for confinement, the reactor essentially acts as a battery, producing an EMF source that may be loaded through the end electrodes in the same way as a hydromagnetic capacitor [17]. Most likely, the optimum design would just maintain the rotation and the necessary diffusion time.

Many implementations of open systems attempt the direct conversion of charged fusion product energy into electrical energy [18, 19]. A recent suggestion for centrifugal confinement devices [20] captures α particle energy both through the centrifugal potential (which goes directly into the rotation energy) and through a retarding potential (direct conversion). But the amount of energy extractable to maintain the rotation energy is a small fraction of the alpha particle kinetic energy. In contrast, as proposed here, the α particle energy can be almost entirely converted into potential energy.

The fact that the effect proposed here acts volumetrically – not at a surface – may importantly alleviate the major engineering hurdle facing the use of rotating mirrors as fusion reactors [1, 2, 5], namely the endplate electrodes. These electrodes need to support large electric fields which are subject to breakdown.

Because centrifugal fusion reactors are run in the hot ion mode ($T_i > T_e$), the alpha channeling effect is particularly fitting. The ions are hot in rotating plasma because they are born at the rotation speed. The rapid removal of α particles, which are slowed down primarily by electrons, then removes an important electron heat source, thereby permitting an even cooler electron temperature. The cooler electron temperature in turn gives rise to a lower ambipolar potential, which means higher ion confinement. Higher ion confinement, in turn, then relaxes the need for additional rotational confinement, so that the mirror can be operated at lower rotation speeds and lower plasma potential. In addition to reducing the electron heating, the quick expulsion of alpha particles by waves reduces the dilution of fuel ions by the alpha particles. Like in a conventional mirror reactor, where the alpha particle ash can dilute the fuel by as much as 30% [21], the prompt removal of this ash (and the chan-

neling of that energy to fuel ions) can increase greatly the effective fusion reactivity at fixed plasma pressure [12].

In conclusion, we generalized the alpha channeling effect to rotating plasma. A new quantity that appears in rotating plasma is the branching ratio, which measures the amount of particle kinetic energy that flows into particle potential energy as compared to the amount which flows into wave energy. By arranging for sufficient channeling of fusion alpha energy directly into electric potential energy, the rotation of the plasma can be maintained against momentum loss. The prompt removal of alpha particles also increases the effective fusion reactivity. Also, the volumetric maintenance of the radial potential should reduce the engineering complexity of the technologically challenging mirror endplates, if not to eliminate the need for these plates entirely. Moreover, the alpha channeling effect is particularly well matched to enhance the reactor prospects of centrifugal fusion reactors, since these reactors are imagined to operate best at low electron temperature and high ion temperature. While the channeling of alpha energy in rotating plasma appears to significantly enhance the prospects for controlled nuclear fusion through centrifugal confinement, it remains to identify the specific plasma waves that can accomplish the speculative concepts put forth here.

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